To Infinity and Beyond

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Close to Infinite?

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OR

2

Thinking about Infinity

- What is ∞? Is it a number?
- How big is it?
- Is there anything bigger?
- What is $\infty - \infty$?
- Are any numbers close to ∞?
- Is anything actually infinite?

What is Infinity?

Let's start with an easier question:

What is 2?

Counting something that never ends

Whenever you have a collection (or set) of objects, you can ask how many objects there are in the collection.

What about the set of Natural Numbers: $\{1, 2, 3, 4, \ldots\}$? How many objects are there?

Answer:

$$\infty$$

What else is infinite?

Natural Numbers:

$$\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15, \ldots\}$$

Non-negative Integers:

$$\{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15, \ldots\}$$

Even Natural Numbers:

$$\{\ 2,\ 4,\ 6,\ 8,\ 10,\ 12,\ 14, \ldots\}$$

Integers:

$$\{\ldots,\ -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5, \ldots\}$$

Rational Numbers:

$$\left\{\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \frac{3}{1}, \frac{1}{4}, \frac{3}{4}, \frac{4}{3}, \ldots\right\}$$

Real Numbers:

Which is larger?

$$\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16, \ldots\}$$

$$\{\quad 2,\quad 4,\quad 6,\quad 8,\quad 10,\quad 12,\quad 14,\quad 16, \ldots\}$$

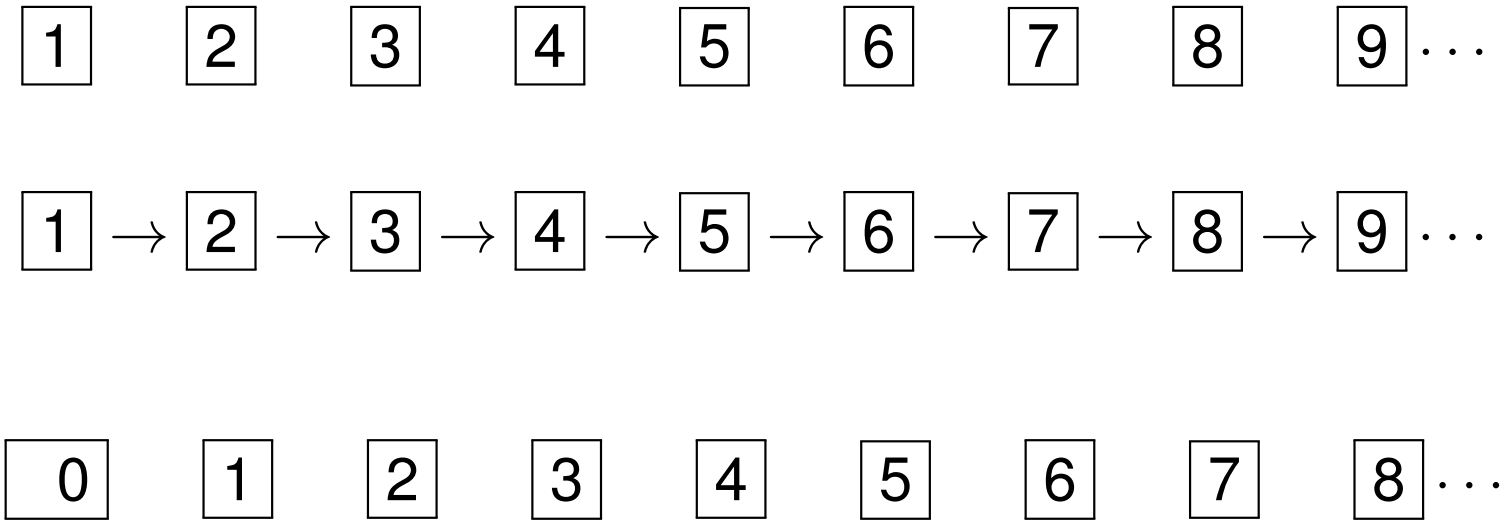
Are there more natural numbers than even numbers? Twice as many?

Comparing sizes

If you have a bunch of umbrellas, and a bunch of boxes, how can you tell whether you have more umbrellas or more boxes?

Naturals vs. Non-negative Integers

A motel with infinitely many rooms (room 1, room 2, room 3, etc.) is currently full. If a new traveler arrives, is it possible to give him a room?



Naturals vs. Evens vs. Integers

Can we put every even number into a room, with no rooms left over?

Can we put every integer (positive or negative whole number) into a room with no rooms left over?

Naturals vs. Rationals

Can we put every rational number (fraction) into its own room (with no rooms left over)?

$$\boxed{\frac{1}{1}}\ \boxed{\frac{1}{2}}\ \boxed{\frac{2}{1}}\ \boxed{\frac{1}{3}}\ \boxed{\frac{3}{1}}\ \boxed{\frac{2}{3}}\ \boxed{\frac{3}{2}}\ \boxed{\frac{1}{4}}\ \boxed{\frac{4}{1}}\ \boxed{\frac{3}{4}}\ \boxed{\frac{4}{3}}\ \ldots$$

Conclusion (so far)

The evens, non-negative integers, integers, and rationals all have the same size: the size of the natural numbers.

What about the real numbers?

There are a LOT of real numbers

Is it possible to give every real number its own room?

Room #:	Real Number:
1	$0.\mathbf{3}1542621032$
2	$2.0\mathbf{1}484729067\ldots$
3	$0.12\mathbf{3}4567891011\ldots$
4	$3.141\mathbf{5}926535898\ldots$
5	$0.3333\mathbf{3}33333333\ldots$
$\vdots$	$\vdots$

A Missing Number: $0.\mathbf{55575}\ldots$

Infinity is weird

There are more real numbers than natural numbers!!!!!

There are different sizes of infinity!!!!!!!!!!!!!

The best thing: the more you investigate infinity, the weirder it gets!