Controlling Domination in Infinite Graphs

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Joint work with Matthew Jura and Tyler Markkanen

Dominating Sets and Domatic Number

The domatic number $d(G)$ of a graph is the size of the largest partition of vertices into dominating sets.

A set D of vertices is dominating if every vertex not in D is adjacent to a vertex in D.

Goal

Motivation: Computability Theory.

We need to control colors of distant vertices.

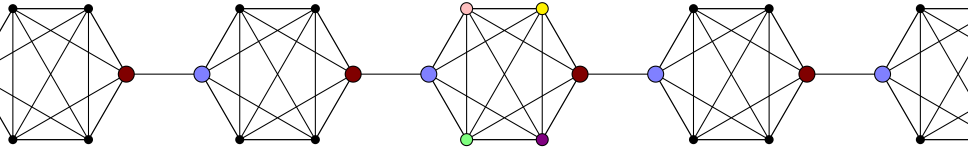
Question

Is there an infinite regular graph containing an infinite set of vertices which must belong to the same dominating set in any domatic n-partition?

Yes

Theorem

There an infinite regular graph containing an infinite set of vertices which must belong to the same dominating set in any domatic 6-partition.

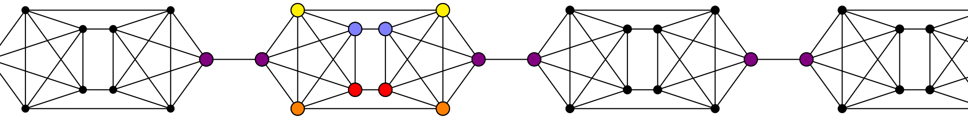


Total Domatic Partitions

A set D of vertices is total dominating if every vertex ~~not in D~~ is adjacent to a vertex in D.

Theorem

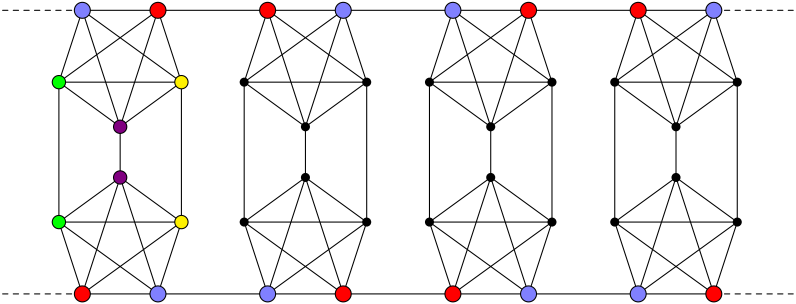
There an infinite regular graph containing an infinite set of vertices which must belong to the same total dominating set in any total domatic 5-partition.



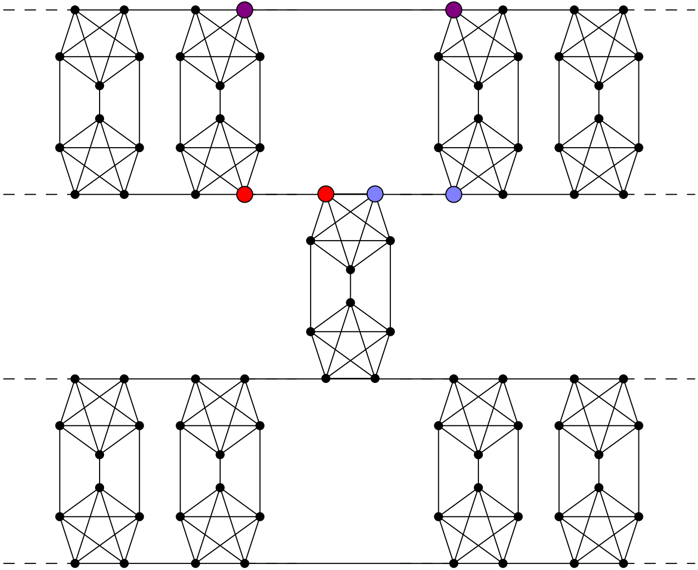
A Second Disjoint Set

Our first example (non-total) actually had two disjoint monochromatic sets (but colored differently than each other).

Can we get this for total domatic partitions?



A Second Disjoint Set



Smaller Partitions

A graph with $d(G) = n$ has domatic k-partitions for all $k \leq n$.

Theorem

If $d(G) = n \geq 3$, then for any vertices u and v of G, there is a domatic $(n-1)$-partition in which u and v are colored identically, and a domatic $(n-1)$-partition in which u and v are colored differently.

Question

Is there anything we can control in any way about smaller domatic partitions?

Thanks

Slides and preprint:

math.oscarlevin.com/research.php